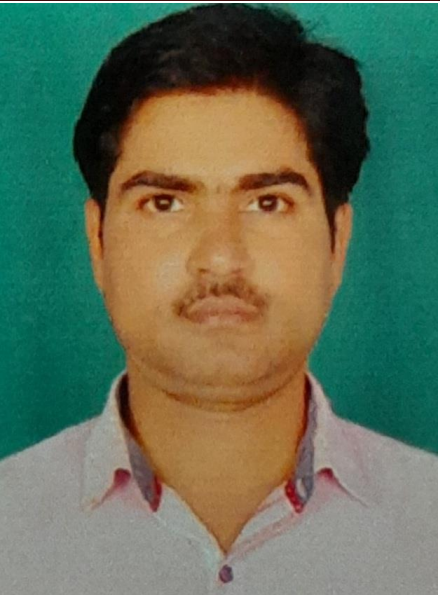




Graph Theory

Class-BCA IV Semester



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OUTLINE-

UNIT :- I

GRAPH THEORY

History of graph theory

Basics Concept Of Graph theory

Why Graph Theory ?



- Graphs used to model pair wise relations between objects
- Generally a network can be represented by a graph
- Many practical problems can be easily represented in terms of graph theory

Graph Theory - History

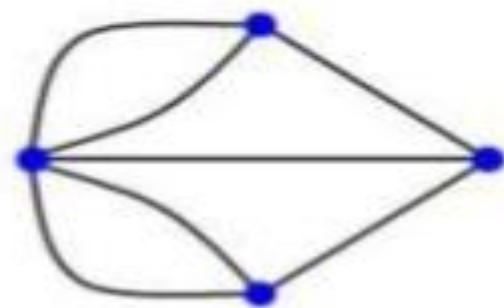
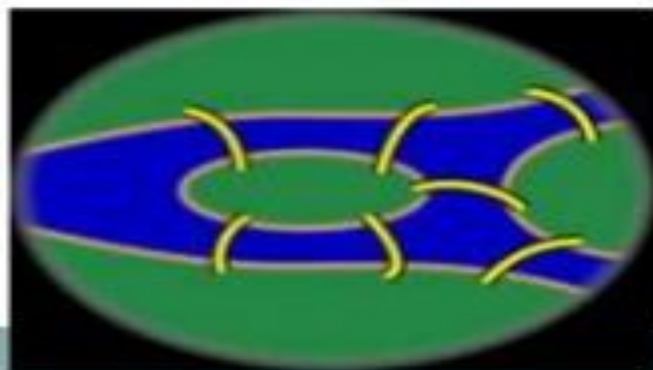


The origin of graph theory can be traced back to Euler's work on the Königsberg bridges problem (1735), which led to the concept of an Eulerian graph. The study of cycles on polyhedra by Thomas P. Kirkman (1806 - 95) and William R. Hamilton (1805-65) led to the concept of a Hamiltonian graph.

Graph Theory - History

- Begun in 1735
- Mentioned in Leonhard Euler's paper on "*Seven Bridges of Königsberg*".

Problem : Walk all 7 bridges without crossing a bridge twice



Graph Theory – History.....

Cycles in Polyhedra - polyhedron with no Hamiltonian cycle



Thomas P. Kirkman



William R. Hamilton



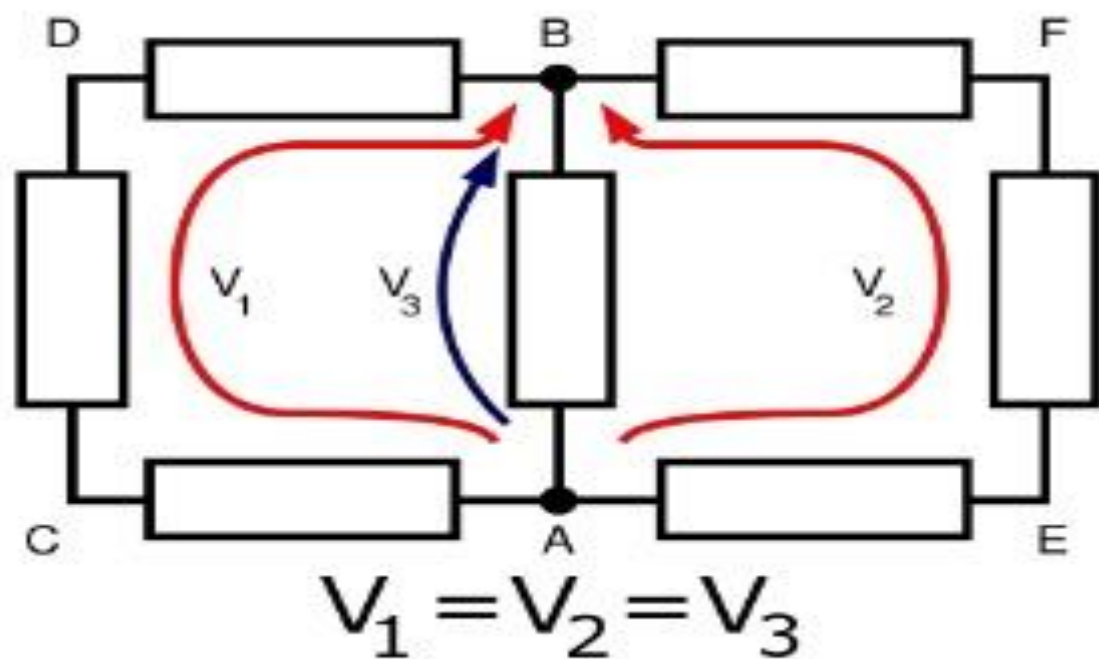
Hamiltonian cycles in Platonic graphs

Graph Theory – History.....

Trees in Electric Circuits



Gustav Kirchhoff



Basic Concepts of Graph Theory



Definition: Graph



- A graph is a collection of nodes and edges
- Denoted by $G = (V, E)$.

$V = \mathbf{nodes}$ (vertices, points).

$E = \mathbf{edges}$ (links, arcs) between pairs of nodes.

Graph size parameters: $n = |V|$, $m = |E|$.

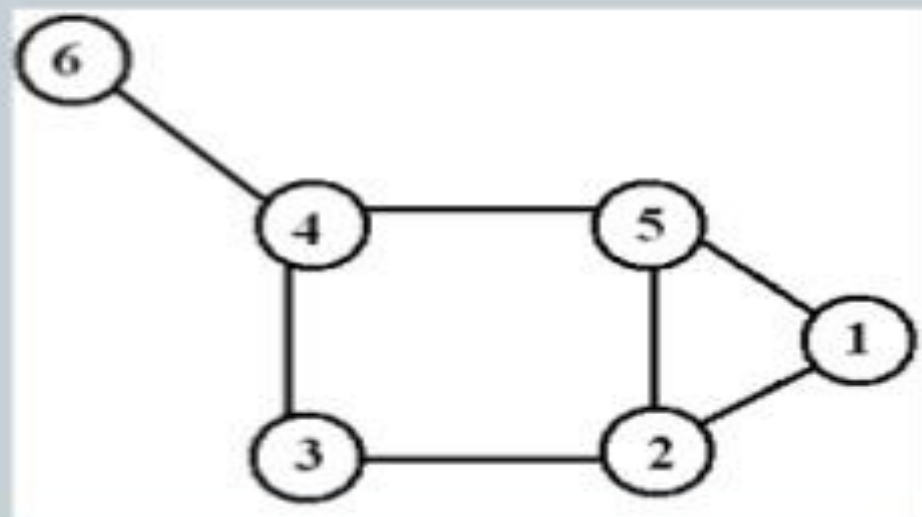


Vertex & Edge



- Vertex /Node
 - Basic Element
 - Drawn as a node or a dot.
 - **Vertex set** of G is usually denoted by $V(G)$, or V or V_G
- Edge /Arcs
 - A set of two elements
 - Drawn as a line connecting two vertices, called end vertices, or endpoints.
 - The edge set of G is usually denoted by $E(G)$, or E or E_G
- Neighborhood
 - For any node v , the set of nodes it is connected to via an edge is called its neighborhood and is represented as $N(v)$

Graph :Example



- $n := 6$, $m := 7$
- Vertices (V) := {1,2,3,4,5,6}
- Edge (E) := {1,2}, {1,5}, {2,3}, {2,5}, {3,4}, {4,5}, {4,6}}
- $N(4) := \text{Neighborhood}(4) = \{6,5,3\}$

Edge types:

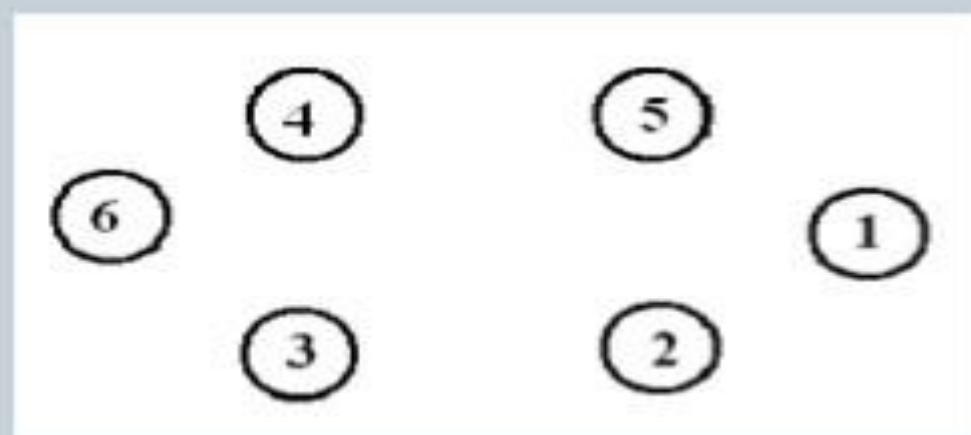


- **Undirected**;
 - E.g., distance between two cities, friendships...
- **Directed**; ordered pairs of nodes.
 - E.g ,...
 - Directed edges have a **source** (head, origin) and **target** (tail, destination) vertices
- **Weighted** ; usually weight is associated .

Empty Graph / Edgeless graph



- No edge

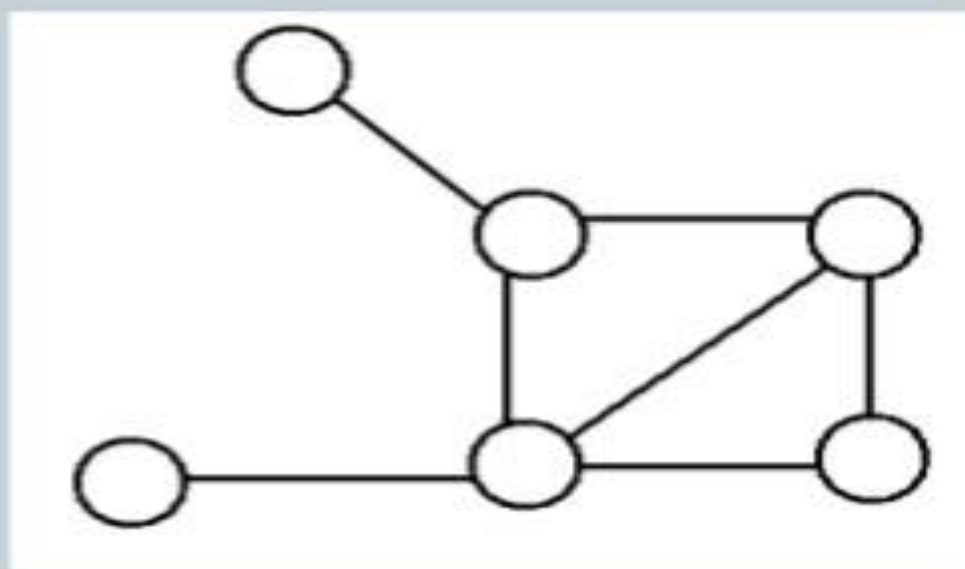


- **Null graph**

- No nodes
- Obviously no edge

Simple Graph (Undirected)

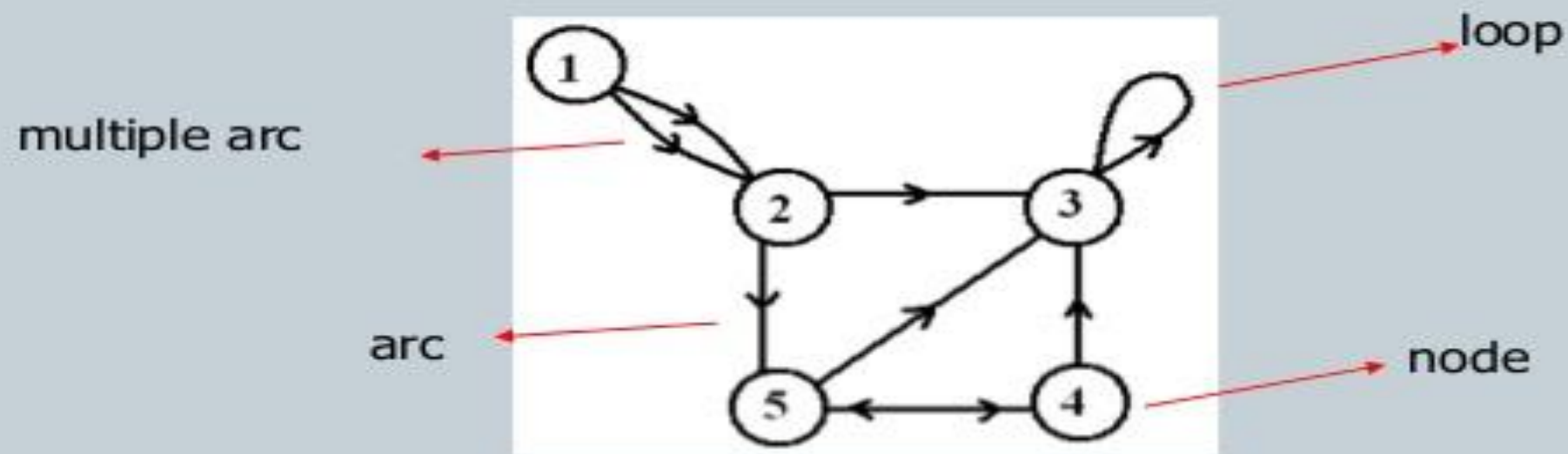
- Simple Graph are undirected graphs without loop or multiple edges
- $A = A^T$



For simple graphs, $\sum_{v_i \in V} \deg(v_i) = 2|E|$

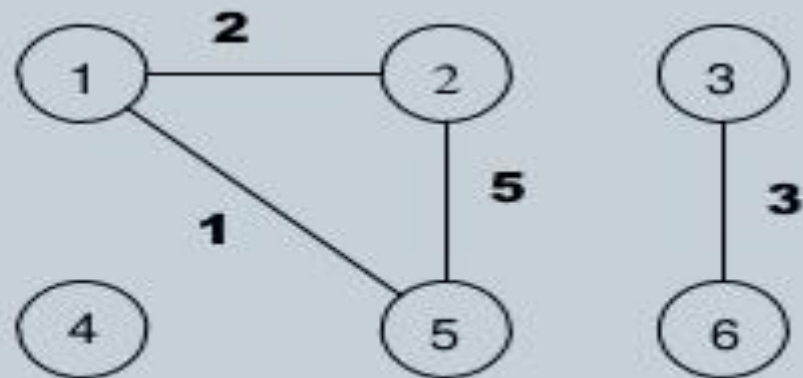
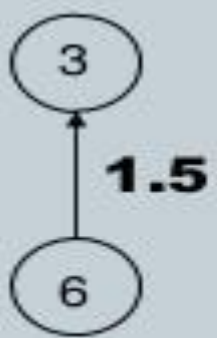
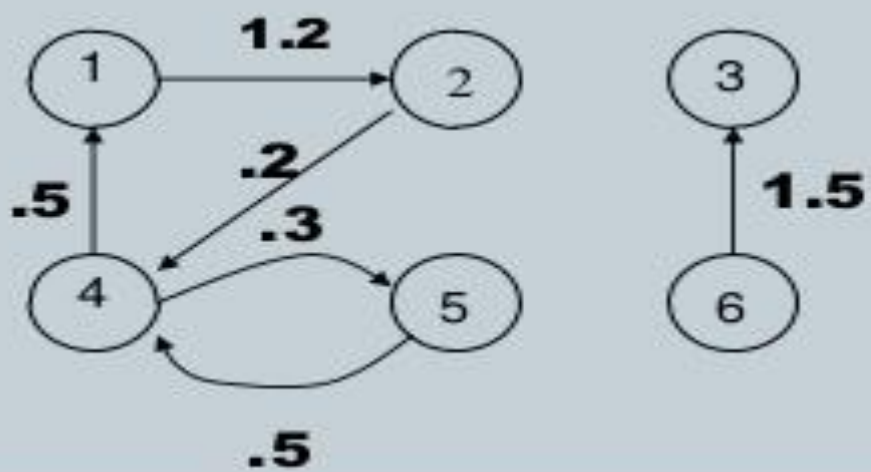
Directed graph : (digraph)

- Edges have directions
- $A \neq A^T$



Weighted graph

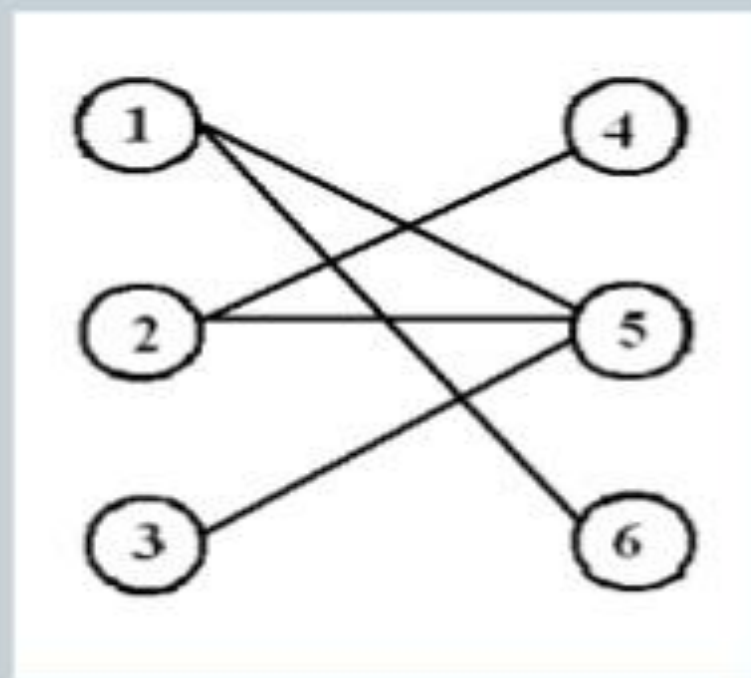
- is a graph for which each edge has an associated *weight*



Bipartite Graph



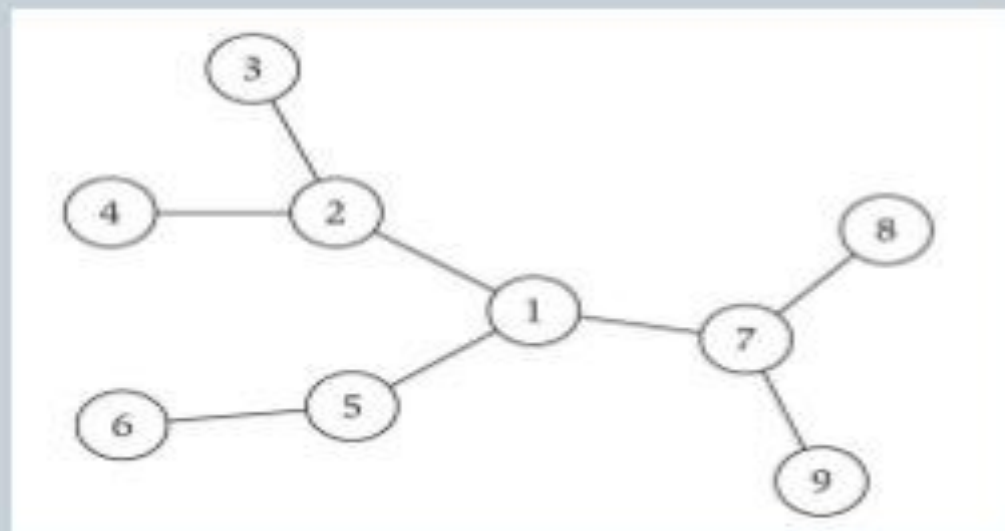
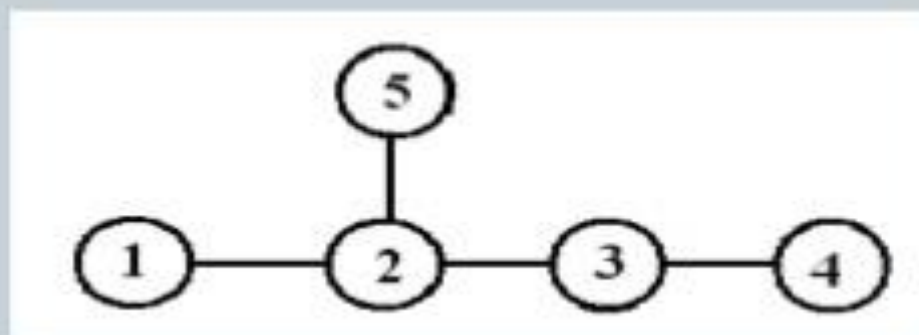
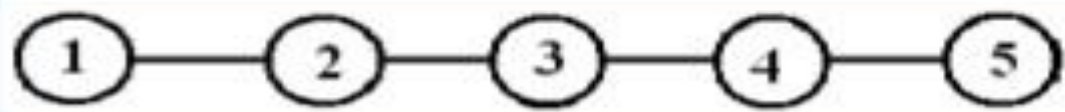
V can be partitioned into 2 sets V_1 and V_2
such that $(u,v) \in E$ implies
either $u \in V_1$ and $v \in V_2$
OR $v \in V_1$ and $u \in V_2$.



Trees



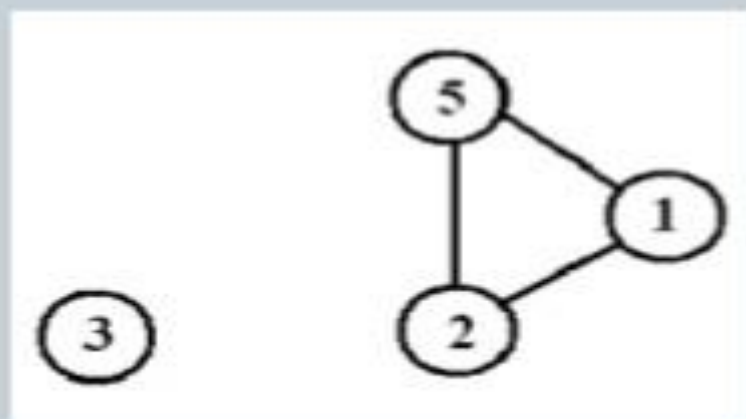
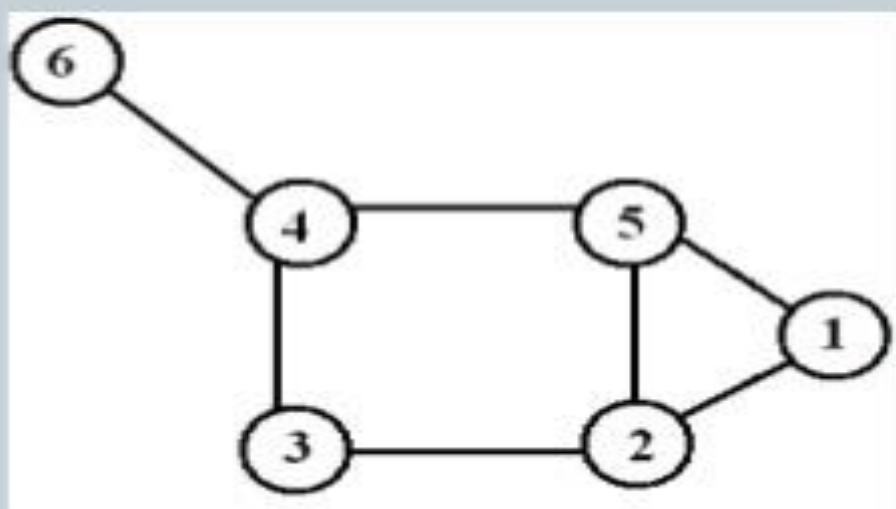
- An undirected graph is a **tree** if it is connected and does not contain a cycle (**Connected Acyclic Graph**)
- Two nodes have *exactly* one path between them



Subgraph



- Vertex and edge sets are subsets of those of G
 - ◉ a *supergraph* of a graph G is a graph that contains G as a subgraph.



QUESTIONS:-

- 1. What is graph theory?**
- 2. What is the basic concept of graph theory?**
- 3. What is the history of graph theory explain it?**

REFERENCES:

**1. Clark J. and Holron D.A. ,{lirsr
LookarCraph ltheory". Allied
publishers. 1995.**

**2. Mott J.L., Kandel A. and Baker T.P.
Discrerc Nlathematics ibr Computer
Scientists and
Mathcnlalicians",Prentice Hall of
India. 1996.**

**Liu C.I,.. "Elcments ol Discrete
,lathcmatics '. Nic (;ra\ \ Hill. 1985.**

**Rosen K.H., "Discrele Mathematics
and lrs Applicarions',. Mc Grau
Hi11.2007.**

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Thanks