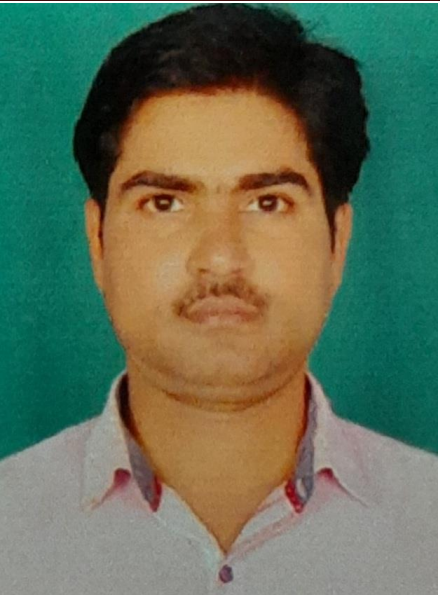




Graph Theory

Class-BCA IV Semester



Dr. Vijay Kant Sharma

Assistant professor ,

Department Of Computer Application

Jagatpur P.G. College, Varanasi

Affiliated to Mahatma Gandhi Kashi
vidhyapith Varanasi

[Email-mzp.vijay@gmail.com](mailto:mzp.vijay@gmail.com)

OUTLINE-

UNIT :- I

Graph Theory

(i) Coloring

(ii) Covering

(iii) Partitioning

Graph Coloring

- Painting all the vertices of a graph with colors such that no two adjacent vertices have the same color – **Proper coloring** (or) **Coloring**
- A graph in which every vertex has assigned a color according to proper coloring – **Properly colored graph.**

[A proper coloring requires the min. no. of colors]



Chromatic Number

- The **chromatic number** of a graph is the smallest number of colors with which it can be colored.

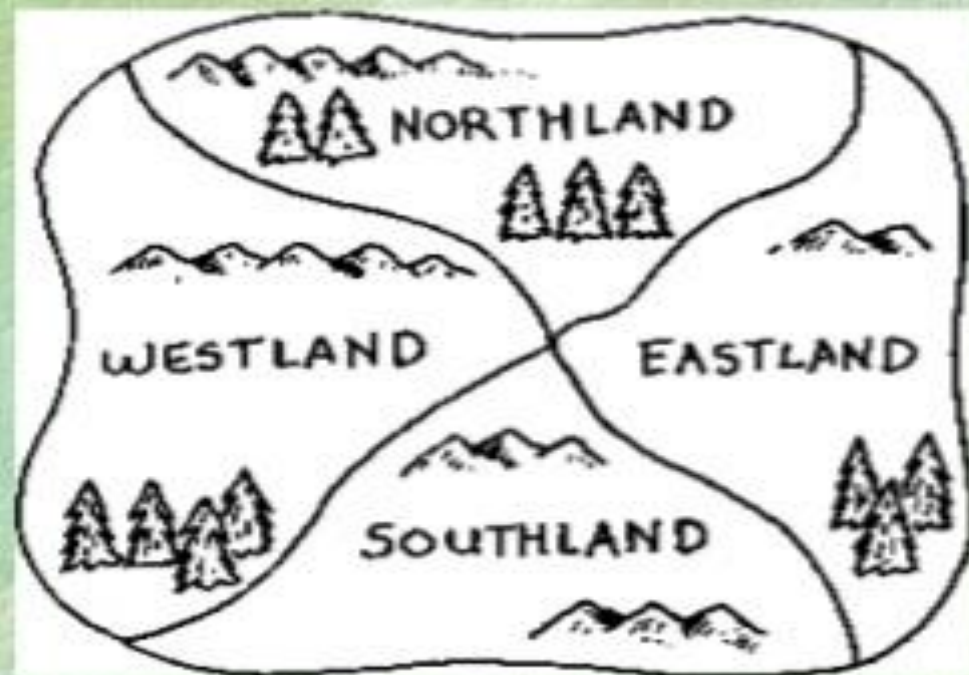
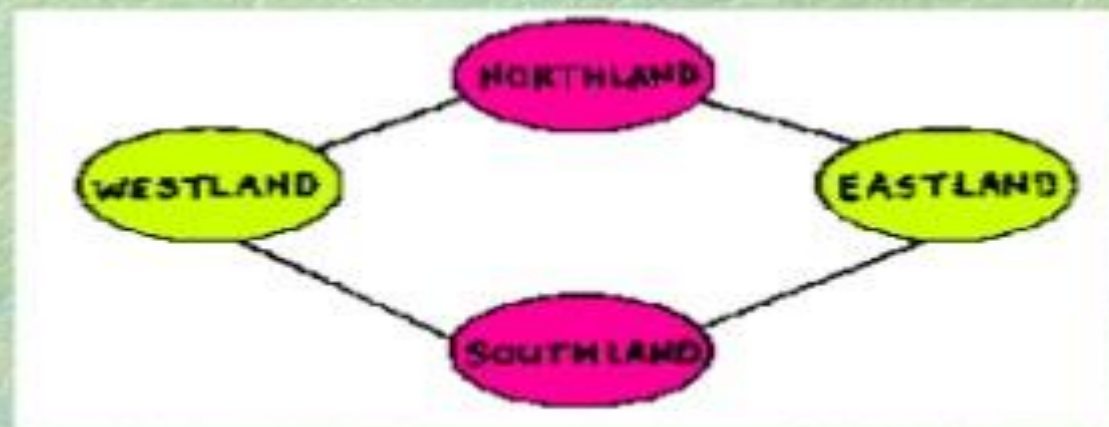
In the example above, the chromatic number is 4.

- The **Chromatic number $\chi(G)$** is the smallest k such that G has proper k -coloring. G is called **k -Chromatic Graph**.

Some Basic Definition:

- A graph consisting of only isolated vertex is **1-Chromatic**.
- A graph with one or more edges is **atleast 2-Chromatic**.
- A complete graph with n vertices is **n -chromatic**.
- A graph with 1 circuit with $n \geq 3$ vertices is **2-chromatic** if n is even and **3-chromatic** if n is odd.
- Every tree is **2-chromatic**.

Try coloring our
Lands map using a
graph:



The graph can be colored
with two colors, just like
the map! The chromatic
number of this graph is 2.

Chromatic Partitioning

- A set of vertices in a graph is said to be an **Independent set** if no two vertices in the set are adjacent.
- The no. of vertices in the largest independent set of a graph G is called the **Independence number.(or) Coefficient of internal stability** denoted by $\beta(G)$
- For k -chromatic graph $\beta(G) \geq n/k$
- In a simple, connected graph G , the Partition of all vertices into smallest possible no. of disjoint independent set - **Chromatic Partitioning**

Chromatic Polynomial

- The graph G of n vertices can be properly colored in many ways using a sufficiently large no. of colors. The property of a graph can be expressed by a polynomial called Chromatic Polynomial.

Definition

- The value of the **Chromatic polynomial** of a graph G with n vertices gives the no. of ways of properly coloring the graph using λ or fewer colors. It is denoted by $P_n(\lambda)$ or $P_G(\lambda)$.

Applications of graph coloring

Sudoku

- Each cell is a vertex
- Each integer label is a “color”
- A vertex is adjacent to another vertex if one of the following hold:
 - Same row
 - Same column
 - Same 3x3 grid
- Vertex-coloring solves Sudoku

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Other Applications...

- Traffic signal design
- Scheduling of exams
- Register allocation
- Biological networks
- Testing printed circuit boards
- Time Tabling
- Channel Assignment(Radio station)

QUESTIONS:-

1. What is the coloring of graph?
2. What is the chromatic number of graph?
3. What is the chromatic polynomial of graph?
4. What is partitioning of graph?

REFERENCES:

- 1. Clark J. and Holron D.A. ,{"lirsr LookarCraph ltheory". Allied publishers. 1995.**
 - 2. Mott J.L., Kandel A. and Baker T.P. Discrerc Nlathematics ibr Computer Scientists and Mathcnlalicians",Prentice Hall of India. 1996.**
- Liu C.I,.. "Elcments ol Discrete \,lathcmatics '. Nic (;ra\\ Hill. 1985.**
- Rosen K.H., "Discrele Mathematics and lrs Applicarions',. Mc Grau Hi11.2007.**

Declaration

The content is exclusively meant for academic purpose and for enhancement teaching and learning. Any other use of economic/commercial purpose is strictly prohibited. The users of the content shall not distribute ,disseminate or share it with anyone else and its use is restricted to advancement of individual knowledge. The information provided in this e-content is authentic and best as per my knowledge.

Dr.Vijay kant Sharma

Assistant professor

Department of computer Application

jagatpur P.G. College Varanasi Affiliated to Mahatma

Gandhi Kashi Vidyapith Varanasi

Thanks