Graph Theory
Class-BCA IV Semester

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OUTLINE-
UNIT-II

Connectivity Of a Graph

Connectivity

- **Definition:** A **path** of length n from u to v, where n is a positive integer, in an **undirected graph** is a sequence of edges $e_1, e_2, \ldots, e_n$ of the graph such that $e_1 = \{x_0, x_1\}$, $e_2 = \{x_1, x_2\}, \ldots, e_n = \{x_{n-1}, x_n\}$, where $x_0 = u$ and $x_n = v$.
- When the graph is simple, we denote this path by its **vertex sequence** $x_0, x_1, \ldots, x_n$, since it uniquely determines the path.
- The path is a **circuit** if it begins and ends at the same vertex, that is, if u = v.

Connectivity

- **Definition (continued):** The path or circuit is said to **pass through** or traverse $x_1, x_2, \ldots, x_{n-1}$.
- A path or circuit is **simple** if it does not contain the same edge more than once.

Connectivity

- Let us now look at something new:

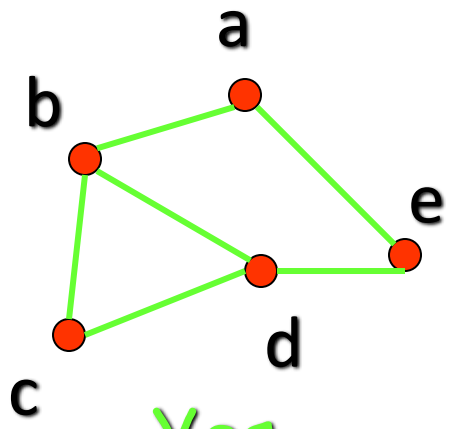
- **Definition:** An undirected graph is called **connected** if there is a path between every pair of distinct vertices in the graph.

- For example, any two computers in a network can communicate if and only if the graph of this network is connected.

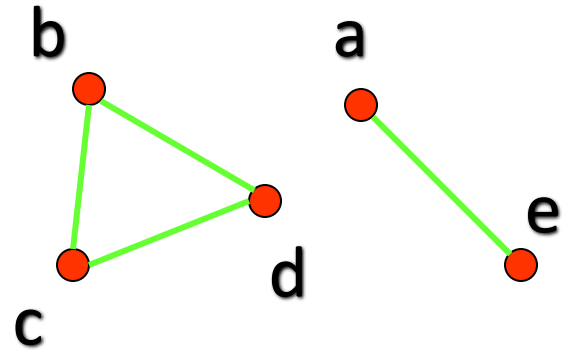
- **Note:** A graph consisting of only one vertex is always connected, because it does not contain any pair of distinct vertices.

Connectivity

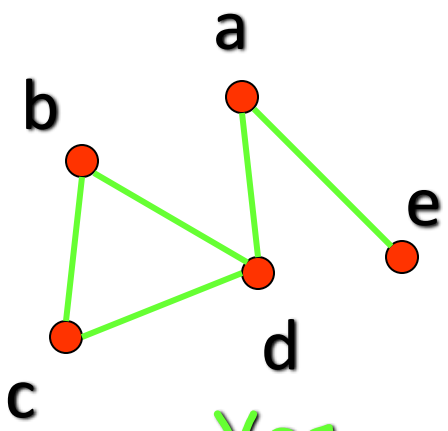
- **Example:** Are the following graphs connected?



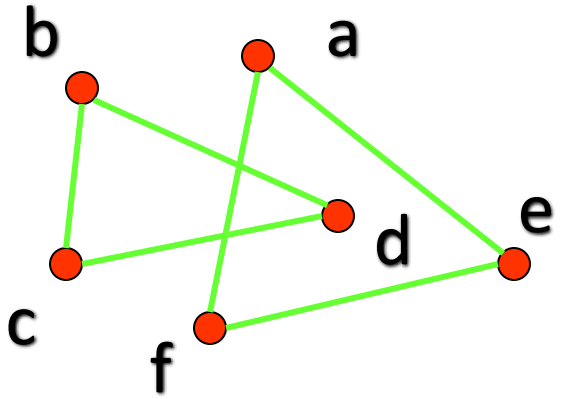
Yes.



No.



Yes.



No.

Connectivity

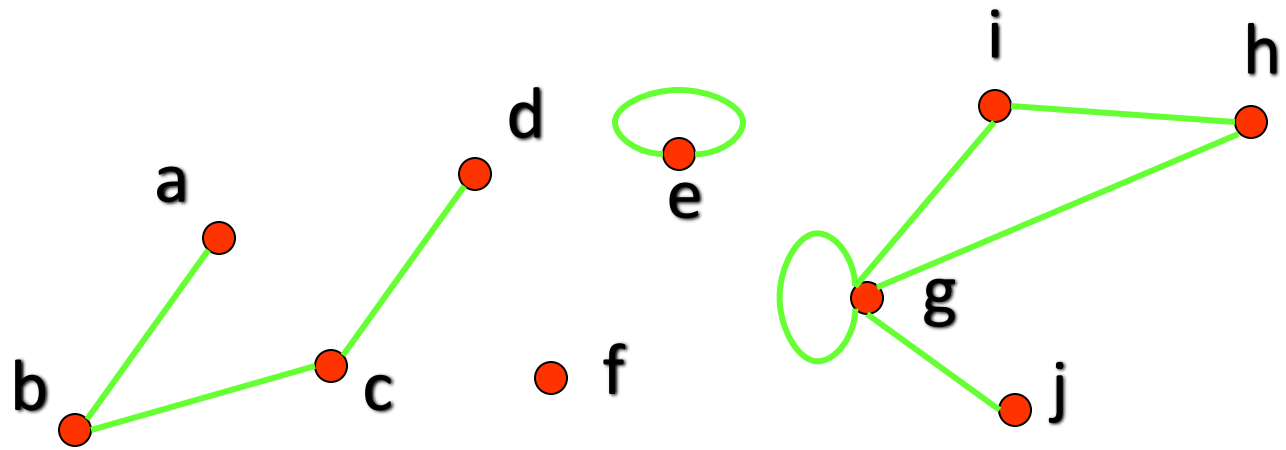
- **Definition:** A graph that is not connected is the union of two or more connected subgraphs, each pair of which has no vertex in common. These disjoint connected subgraphs are called the **connected components** of the graph.

- **Definition:** A **connected component** of a graph G is a maximal connected subgraph of G.

- E.g., if vertex v in G belongs to a connected component, then all other vertices in G that is connected to v must also belong to that component.

Connectivity

•**Example:** What are the connected components in the following graph?



Solution: The connected components are the graphs with vertices {a, b, c, d}, {e}, {f}, {i, g, h, j}.

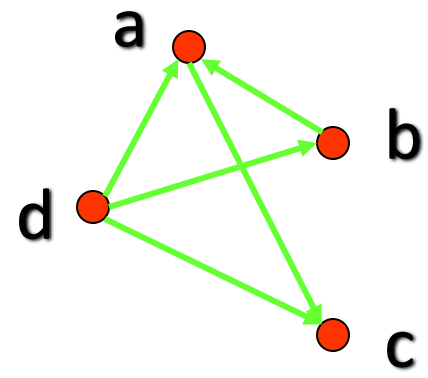
Connectivity

- **Definition:** An directed graph is **strongly connected** if there is a path from a to b and from b to a whenever a and b are vertices in the graph.

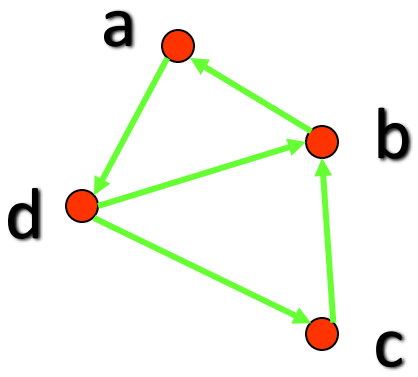
- **Definition:** An directed graph is **weakly connected** if there is a path between any two vertices in the underlying undirected graph.

Connectivity

•**Example:** Are the following directed graphs strongly or weakly connected?



Weakly connected, because, for example, there is no path from b to d.

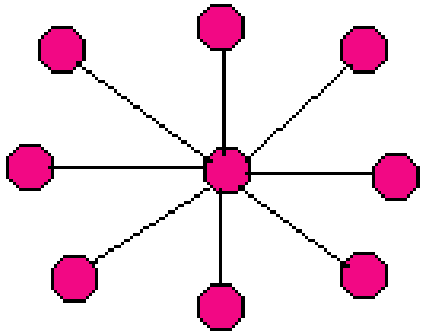


Strongly connected, because there are paths between all possible pairs of vertices.

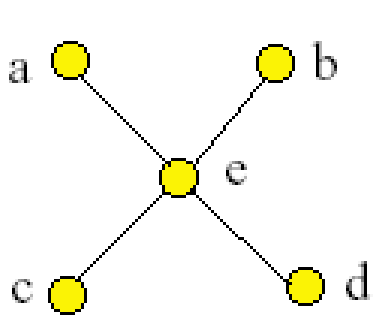
Cut vertices and edges

If one can remove a vertex (and all incident edges) and produce a graph with more components, the vertex is called a ***cut vertex*** or ***articulation point***.

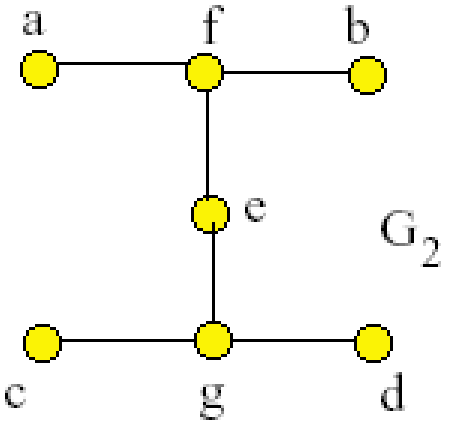
Similarly if removal of an edge creates more components the edge is called a ***cut edge*** or ***bridge***.



$K_{1,8}$

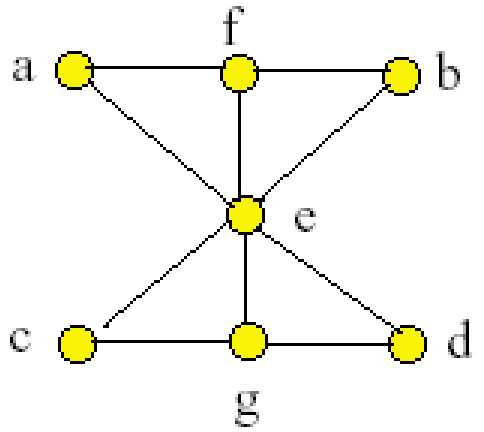


G_1



G_2

In the star network the center vertex is a cut vertex. All edges are cut edges.



$G_1 \cup G_2$

In the graphs G_1 and G_2 every edge is a cut edge.
In the union, no edge is a cut edge.
Vertex e is a cut vertex in all graphs.

QUESTIONS:-

1. **What is connectivity in graph theory?**
2. **What is the edge connectivity and vertex connectivity?**

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Thanks